Self-induced manipulation of biphoton entanglement in topologically distinct modes [1]

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Abstract

Biphoton states have been promising applications in quantum information processing, including quantum communications, quantum metrology, and quantum imaging. The generation and manipulation of biphoton entanglement in topologically distinct modes paves the way in this direction. Here we present a comprehensive method for regulating the topological properties of the system by combining the nonlinearity in waveguides, i.e. nonlinearity in the waveguide coupling materials, and the waveguide lattice structure. Our method enables the generation of topological biphoton states with the injected pump activation on the topologically trivial modes. This is realized with the self-induced manipulations on pump-dependent nonlinear couplings on the defects, which is unable to be realized while there are no such nonlinear couplings. Specifically, by including the nonlinear gain/loss mechanism in the coupling between the nearest neighbor waveguides and the third-order Kerr nonlinearity effect along the waveguides, the injected pump power will be the controllable parameter for the manipulation of the topology in the defect states and the generation of biphoton entanglement states. We also present an experimental proposal to realize our scheme and its generalization in the contemporaneous "active" topological photonics time-bin platforms. Our method can be used in other SSH models with various defect configurations. Our method enables the reusability and versatility of SSH lattice chips and their application for fault-tolerant quantum information processing, promoting the industrialization process of quantum technology.

Motivation

- To generate biphoton states with topological protection, plus feasibility to adjust its spatial configurations.
- To investigate how nonlinearity makes difference in a simple 1D topological model.

Basic

- **The Su-Schrieffer-Heeger model with defects [2].** For a one-dimensional SSH lattice with $2n$ sites, the Hamiltonian of Su-Schrieffer-Heeger (SSH) model can be written as

$$H = \sum_{j=0}^{n-1}\left(u a_{2j+1}^{\dagger} a_{2j} + \text{H.c.}\right) + \sum_{j=1}^{n-1}\left(v a_{2j}^{\dagger} a_{2j-1} + \text{H.c.}\right) \quad (1)$$

The localized edge states appear at both ends of the one-dimensional chain when $u < v$.

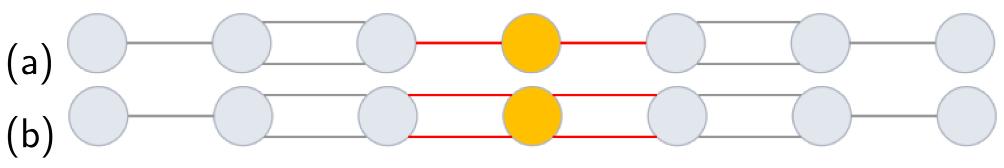

Figure 1: *Different configurations of SSH with defects. (a) Long-long defect (b) Short-short defect.*

Nonlinearity in the waveguide chips

- **Nonlinearity**
  1. Kerr-like nonlinearity [3] determined by the density of the two neighboring sites around the hopping;
  2. The spontaneous four-wave mixing procedure (SFWM) for the generation of biphotons.

  For the waveguide array with $2N+1$ sites, and with the defect placed at site 0 as shown in Fig. 2, the process can be modeled as a nonlinear process in the Hamiltonian:

$$H_{\text{nonlinear}} = \gamma \sum_{j=-N}^{N} (a_j^{p})^2 a_j^{s\dagger} a_j^{i\dagger} + \text{H.c.} \quad (2)$$

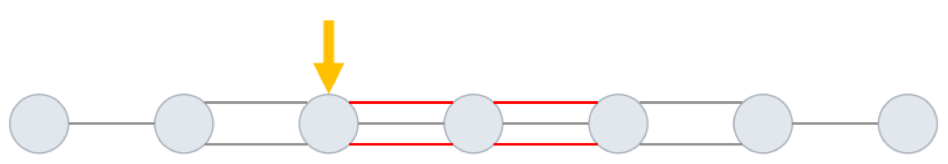

Figure 2: *SSH chain with defect and nonlinearity couplings around defect.*

- **Pump in the SSH model with long-long defects**. The evolution of the pump light is mainly determined by

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\alpha_j^{p}(t) = v_{j-1}^{p}\alpha_{j-1}^{p}(t) + v_j^{p}\alpha_{j+1}^{p}(t) \quad (3)$$

where $j \in [-N, N] := \{-N, -N+1, \cdots, N\}$, and the periodic boundary is imposed at $j=-N$ and $j=N$. Here α^{p} is the amplitude for the pump; v^{p} is the hopping strength with $v_j^{p} = v_{\text{long}}^{p}$ for $j \in \mathsf{L} := \{-3, -5, \cdots\} \cup \{2, 4, \cdots\}$, which describes the linear hopping between waveguides with a long separation; $v_j^{p} = v_{\text{short}}^{p}$ for $j \in \mathsf{S} := \{-2, -4, \cdots\} \cup \{1, 3, \cdots\}$, which gives the linear hopping between waveguides with a short separation; and for $j \in \mathsf{N} := \{-1, 0\}$

$$v_j^{p} = v_{\text{long}}^{p} + \nu^{p}\left(\left|\alpha_{j+1}^{p}(t)\right|^2 + \left|\alpha_j^{p}(t)\right|^2\right) \quad (4)$$

with ν^{p} the nonlinear coefficient for the hopping between waveguides filled with the nonlinear material.

- **Evolution of the signal and idler lights**. Using the classical approximation for the pump light, the corresponding Hamiltonian is

$$\hat{H}_{\text{is}} = \sum_{j=-N}^{N}\left[v_j^{s}\hat{a}_j^{s}\hat{a}_{j+1}^{s\dagger} + v_j^{i}\hat{a}_j^{i}\hat{a}_{j+1}^{i\dagger} + \gamma\left(\alpha_j^{p}\right)^2 \hat{a}_j^{s\dagger}\hat{a}_j^{i\dagger} + \text{H.c.}\right] \quad (5)$$

Similar to the setup for the pump light, here

$$v_j^{s/i} = \begin{cases} v_{\text{long}}^{s/i} & j \in \mathsf{L} \\ v_{\text{short}}^{s/i} & j \in \mathsf{S} \\ v_{\text{long}}^{s/i} + \nu^{s/i}\left(\left|\alpha_{j+1}^{p}(t)\right|^2 + \left|\alpha_j^{p}(t)\right|^2\right) & j \in \mathsf{N} \end{cases} \quad (6)$$

We decompose this into the hopping part and biphoton generation part from the SFWM process as follows:

$$\hat{H}_{\text{hopping}}^{\text{is}} = \sum_{j=-N}^{N} v_j^{s}\hat{a}_j^{s}\hat{a}_{j+1}^{s\dagger} + v_j^{i}\hat{a}_j^{i}\hat{a}_{j+1}^{i\dagger} + \text{H.c.} \quad (7)$$

$$\hat{H}_{\text{biphoton}}^{\text{is}} = \sum_{j=-N}^{N} \gamma\left(\alpha_j^{p}\right)^2 \hat{a}_j^{s\dagger}\hat{a}_j^{i\dagger} + \text{H.c.} \quad (8)$$

Evolution of pump and biphotons

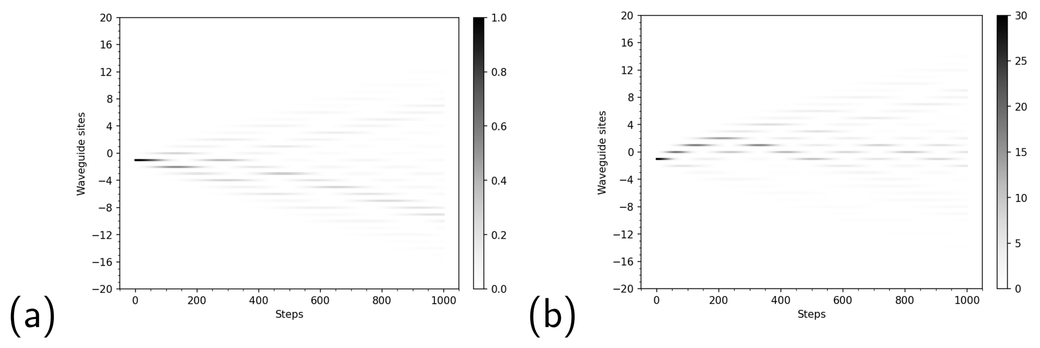

Figure 3: *Pump dynamics in a 103-waveguides chip with a long-long defect centered at the waveguide indexed 0, with different pump power injections: (a) $1\,\mathrm{W}$ and (b) $30\,\mathrm{W}$.*

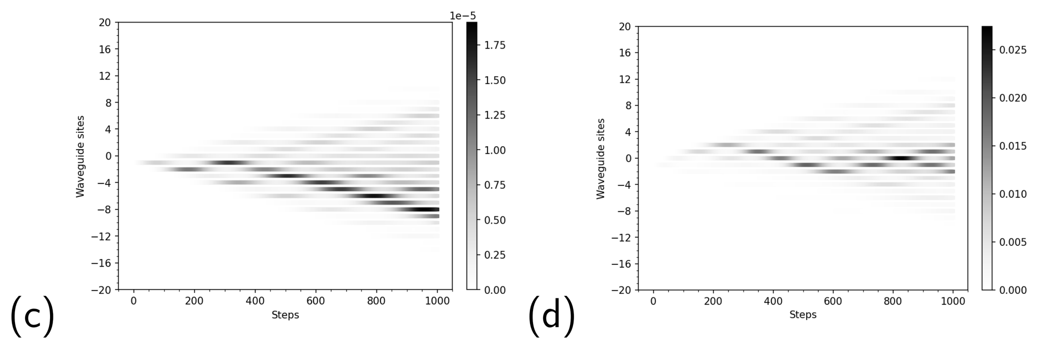

Figure 4: *Biphoton dynamics in a 103-waveguides chip with a long-long defect centered at the waveguide indexed 0, with different pump power injections: (a) $1\,\mathrm{W}$ and (b) $30\,\mathrm{W}$.*

Spectrum analysis

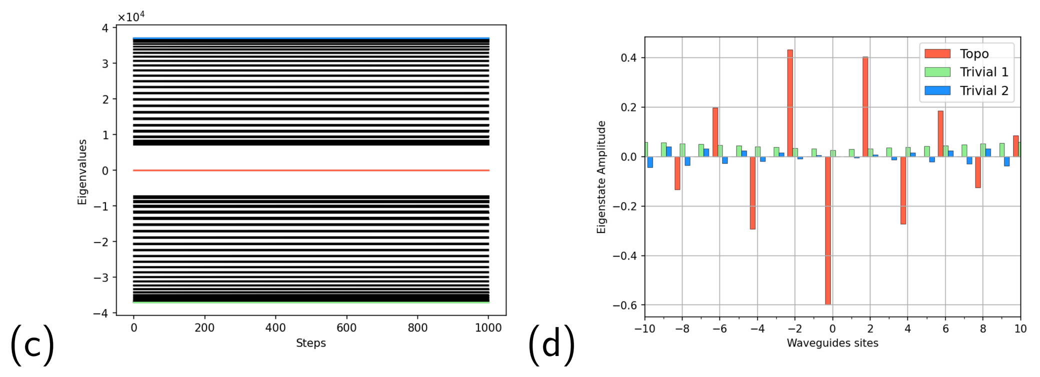

Figure 5: *(a) Spectrum versus steps and (b) eigenvectors at initial step with $1\,\mathrm{W}$ pump.*

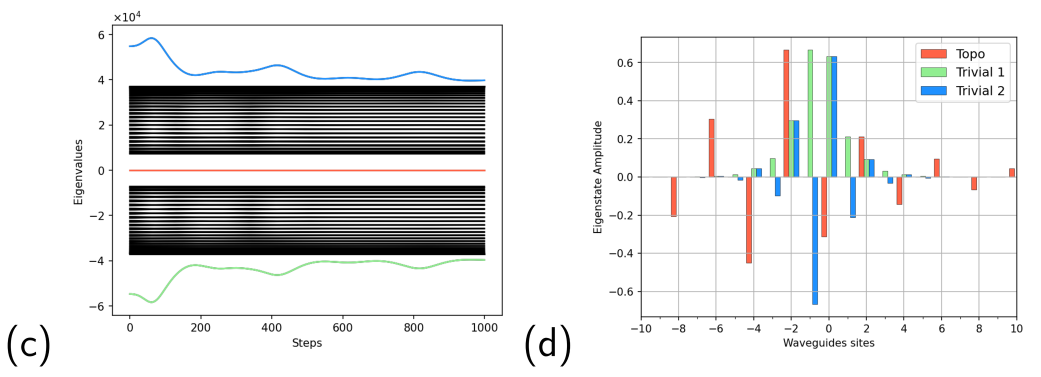

Figure 6: *(a) Spectrum versus steps and (b) eigenvectors at initial step with $30\,\mathrm{W}$ pump.*

Biphoton generation and its robustness

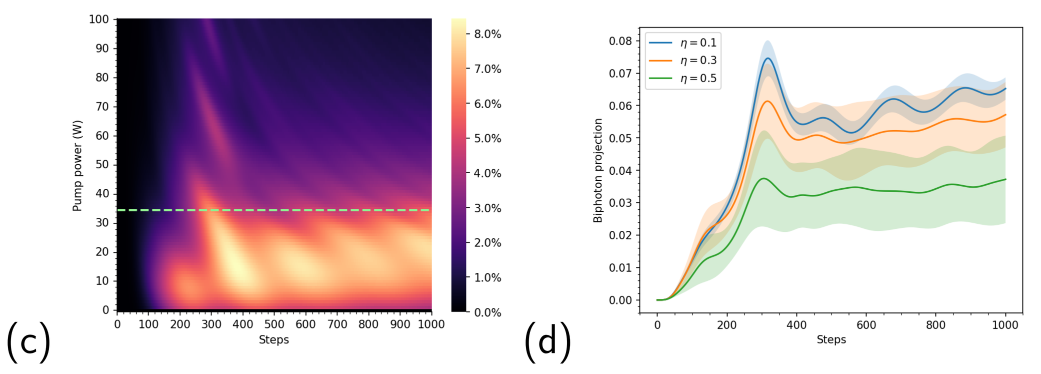

Figure 7: *Heatmap of the topological biphoton weight versus pump power and steps.*

Proposal for the time-bin realization

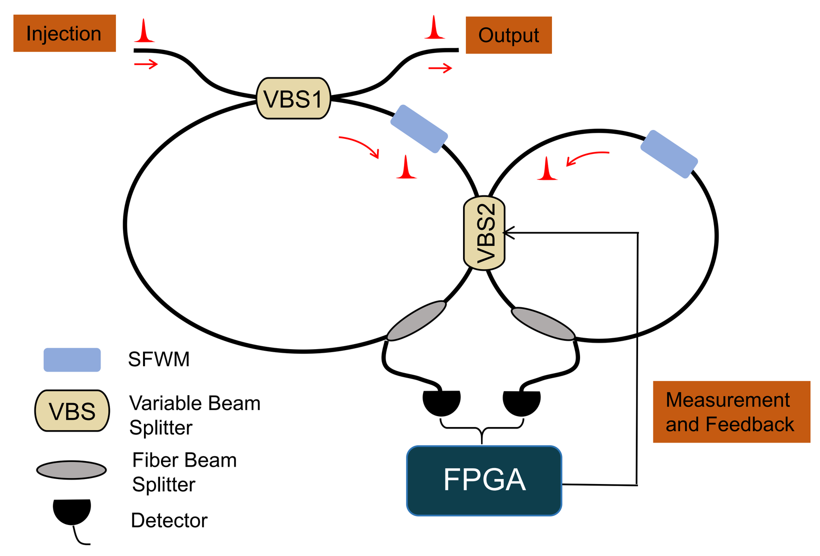

Figure 8: *Actively controlled scheme with time-bin encoding.*

Summary

- By the Kerr-like nonlinear manipulation, we alter the type of the defect in 1D lattice.
- Even injected at the odd site, the light can possess the partial topological protection during the evolution.
- We also provide the feasible proposal in time-bin system.

References

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