Sub-Planck structures and sensitivity of the superposed photon-added or photon-subtracted squeezed-vacuum states [1]

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Abstract

The Wigner function of the compass state (a superposition of four coherent states) develops phase-space structures of dimension much less than the Planck scale $\hbar$, which are crucial in determining the sensitivity of these states to phase-space displacements. In the present work we introduce compasslike states that may have connection to the contemporary experiments, which are obtained by either adding photons to or subtracting photons from the superposition of two squeezed-vacuum states. We show that when a significant quantity of photons is added (or subtracted), the Wigner functions of these states are shown to have phase-space structures of an area that is substantially smaller than the Planck scale. In addition, these states exhibit sensitivity to displacements that is much higher than the standard quantum limit. Finally, we show that both the size of the sub-Planck structures and the sensitivity of our states are strongly influenced by the average photon number, with the photon-addition case having a higher average photon number leading to the smaller sub-Planck structures and, consequently, being more sensitive to displacement than the photon-subtraction case. Our states offer unprecedented resolution to the external perturbations, making them suitable for quantum sensing applications.

Motivation

- To investigate feasible non-Guassian states, of which the phase space shows the chessboard pattern with sub-Planck scale.
- To analyse the various effects on the central interference pattern in the phase space, such as the weight of superposition in SPASVS or SPSSVS, and the average photon number.

Basic

- **Zurek compass state [2].**

$$|\psi\rangle := |x_0/\sqrt{2}\rangle + |{-x_0}/\sqrt{2}\rangle + |\mathrm{i}x_0/\sqrt{2}\rangle + |{-\mathrm{i}x_0}/\sqrt{2}\rangle\,, x_0 \in \mathbb{R}. \quad (1)$$

- **PASVS.** Photon-added squeezed vacuum state

$$\left|\psi^{\pm}_{\mathrm{PA}}\right\rangle := \left(\hat{a}^\dagger\right)^n \hat{S}(r)\,|0\rangle\,. \quad (2)$$

- **PSSVS.** Photon-subtracted squeezed vacuum state

$$\left|\psi^{\pm}_{\mathrm{PS}}\right\rangle := \hat{a}^n \hat{S}(r)\,|0\rangle\,. \quad (3)$$

- **Wigner function of $\hat{\rho}$ [3].**

$$W_{\hat{\rho}}(\boldsymbol{r}) := \mathrm{tr}\left[\hat{\rho}\hat{\Delta}(\alpha)\right],\ \boldsymbol{r} := (x,p)^\top, \quad (4)$$

with $\hat{\Delta}(\alpha) := 2\hat{D}(\alpha)\hat{\Pi}\hat{D}^\dagger(\alpha)$, $\hat{\Pi} := (-1)^{\hat{a}^\dagger\hat{a}}$.

- **Sensitivity to displacement $\delta\alpha$ [4].**

$$O_{\hat{\rho}}(\delta\alpha) := \mathrm{tr}\{\hat{\rho}\hat{D}(\delta\alpha)\hat{\rho}\hat{D}^\dagger(\delta\alpha)\} = \left|\langle\psi|\hat{D}(\delta\alpha)|\psi\rangle\right|^2. \quad (5)$$

Coherent state and Zurek compass state

- **Wigner function of coherent state $|\alpha\rangle$.**

$$W_{|\alpha\rangle}(\beta) = \frac{1}{\pi}\exp\left(-2|\alpha-\beta|^2\right). \quad (6)$$

- **Sensitivity of coherent state $|\alpha\rangle$.**

$$O_{|\alpha\rangle}(\delta\alpha) = \mathrm{e}^{-|\delta\alpha|^2}. \quad (7)$$

  - The smallest noticeable displacement is above the Planck scale, i.e., $|\delta\alpha| > 1$
  - Increasing $\langle\hat{N}\rangle$ does not improve the sensitivity.

$$\left\langle\hat{N}\right\rangle = \left\langle\hat{a}^\dagger\hat{a}\right\rangle = |\alpha|^2 \quad (8)$$

- **Wigner function of compass state.**

$$W_{|\psi\rangle}(\boldsymbol{r}) = W_{\cap}(\boldsymbol{r}) + W_{\sim}(\boldsymbol{r}) + W_{\boxplus}(\boldsymbol{r}), \quad (9)$$

  - $W_{\cap}(\boldsymbol{r})$: Gaussian lobes of four coherent states
  - $W_{\sim}(\boldsymbol{r})$: Gaussian-modulated interference between pair of coherent states
  - $W_{\boxplus}(x,p)$: Chessboard pattern around the origin as

$$\frac{1}{2}\exp\left(-x^2-y^2\right)\left[\cos(2x_0x)+\cos(2x_0p)\right] \quad (10)$$

  - The areas of the chessboard-like pattern is proportional to x_0^{-2}, which is below the Planck scale for $x_0 \gg 1$.

- **Sensitivity of compass state.**
  For $x_0 \gg 1$ and $\delta\alpha \ll 1$,

$$O_{|\psi\rangle}(\delta\alpha) = \frac{1}{4}\mathrm{e}^{-\frac{1}{2}|\delta\alpha|^2}\left[\cos\left(x_0\delta_x\right)+\cos\left(x_0\delta_p\right)\right]^2, \quad (11)$$

  - Sensitivity $\sim x_0^{-1}$

$$\delta_x \pm \delta_p = \frac{2m+1}{x_0}\pi,\ m \in \mathbb{Z}. \quad (12)$$

  - Increasing $\langle\hat{N}\rangle$ increases sensitivity.

$$\langle\hat{N}\rangle = x_0^2/2 \quad (13)$$

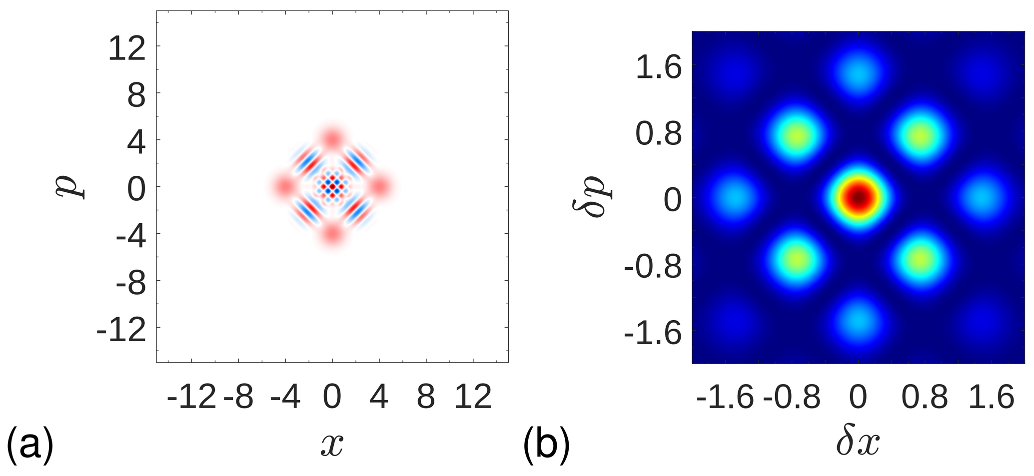

Figure 1: *(a) Wigner distribution and (b) sensitivity of the Zurek compass state* (1) *with* $x_0 = 4$.

Sub-Planck structures in SPASVS

- **SPASVS.** Superposition of PASVSs

$$|\psi_{\mathrm{SPA}}\rangle = c_1\,|\psi^{+}_{\mathrm{PA}}\rangle + c_2\,|\psi^{-}_{\mathrm{PA}}\rangle\,. \quad (14)$$

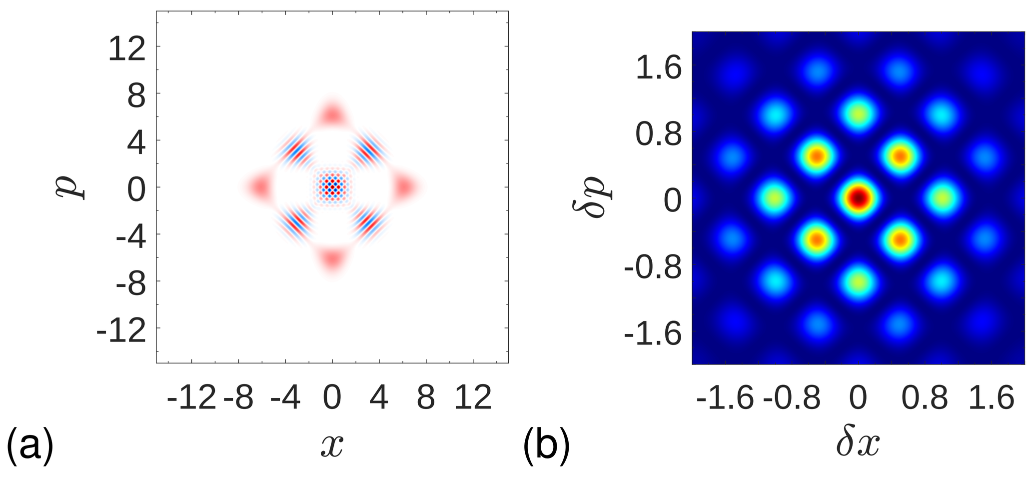

Figure 2: *(a) Wigner distribution and (b) sensitivity of the SPASVS with* $c_1 = c_2$, $r = 0.5$ *and* $n = 10$.

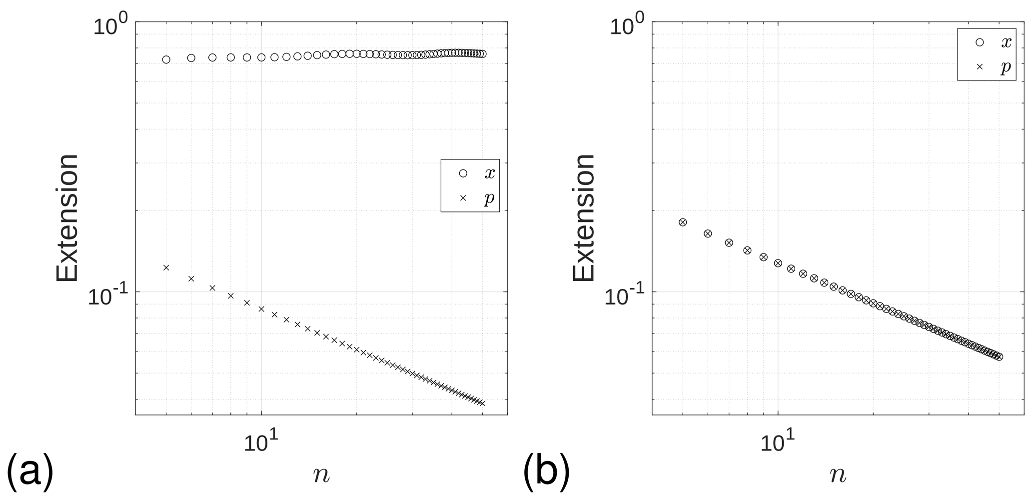

Figure 3: *Extension of the central phase-space structure vs the photon number n of the SPASVS state with* n *chosen from 5 to 50 for (a)* $c_1 = 1/10$, *(b)* $c_1 = 1/\sqrt{2}$.

Sub-Planck structures in PSSVSs

- **SPSSVS.** Superposition of PSSVSs

$$|\psi_{\mathrm{SPS}}\rangle = c_1\,|\psi^{+}_{\mathrm{PS}}\rangle + c_2\,|\psi^{-}_{\mathrm{PS}}\rangle\,. \quad (15)$$

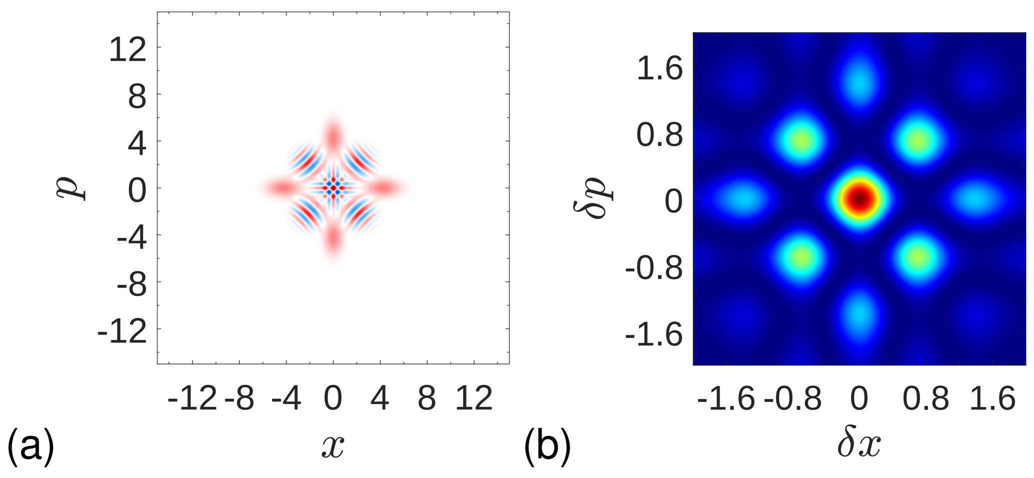

Figure 4: *(a) Wigner distribution and (b) sensitivity of the SPSSVS with* $c_1 = c_2$, $r = 0.5$ *and* $n = 10$.

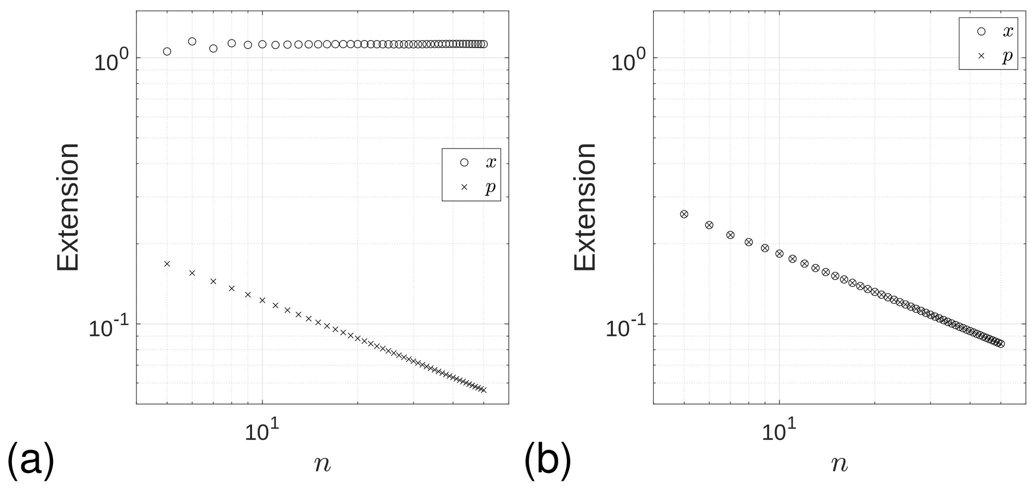

Figure 5: *Extension of the central phase-space structure vs the photon number n of the SPSSVS state with* n *chosen from 5 to 50 for (a)* $c_1 = 1/10$, *(b)* $c_1 = 1/\sqrt{2}$.

Summary

- We investigate the Wigner function of the superpositions of PASVSs and PSSVSs, and demonstrate that these states contain the sub-Planck structure as that in a compass state.
- These states exhibit sensitivity to displacement far below than the standard quantum limit.
- We provide the origin of the sub-Planck structure in our instances by analysing the effects of different weights on states
- We present a novel viewpoint on how the manifestation of the sub-Planck structure is impacted by the addition and/or subtraction of photons and the superposition weight.

References

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