Continuous-time quantum walk on Chimera graph

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1. Introduction

- In the quest to build a programmable quantum simulators with the capability to outperform classical hardware, a careful design is of key importance. The underlying architecture for embedding benchmark problems in the D-Wave quantum annealer is Chimera topology.
- We study continuous-time quantum walk on the planar Chimera graph, Chimera-with-additional-connections and the weak-strong cluster graph.
- The characterization and comparison of these graphs show that the walker's evolution strongly depends upon the graph connectivity, the initial position and initial state of the walker.
- This work can provide a useful insight about minimal modification in Chimera graph connectivity that can maximally impact the computational performance.

2. Continuous-time quantum walks

- The evolution of the walker in continuous-time quantum walks (CTQW) is described by Schrödinger equation

$$i\frac{d}{dt}|j\rangle = \hat{H}|j\rangle, \tag{1}$$

where $\hbar = 1$. The evolution of CTQW is described by a $N \times N$ Hamiltonian $\hat{H}$, with elements

$$H_{ab} = \begin{cases} -\gamma & a \neq b, a \text{ and } b \text{ are connected by an edge } E \\ 0 & a \neq b, a \text{ and } b \text{ are not connected} \\ k\gamma & a = b, k \text{ is the valence of vertex } a, \end{cases} \tag{2}$$

where γ is the transmission rate.
- The transition amplitude $\alpha_{k,j}(t)$ from state *j* at an initial time $t = 0$ to state *k* at time t is

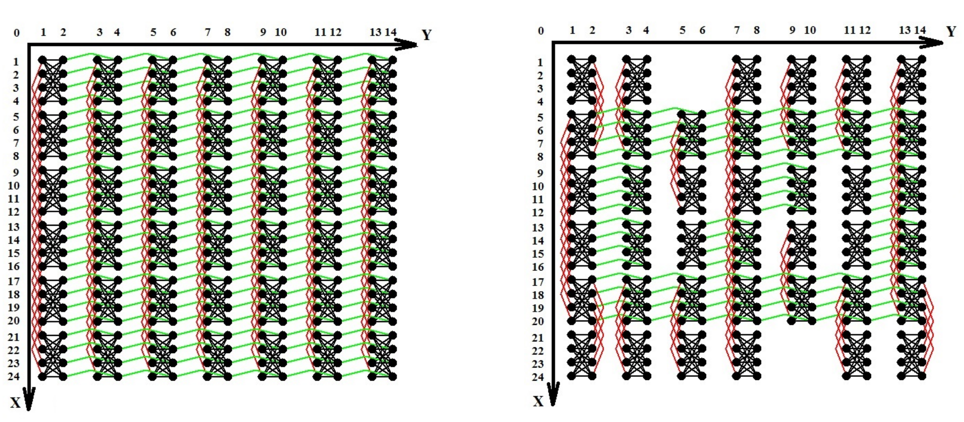
$$\alpha_{k,j}(t) = \langle k|e^{-i\hat{H}t}|j\rangle. \tag{3}$$

- The limiting probability gives the long time average of $\pi_{k,j}(t) \equiv |\alpha_{k,j}(t)|^2$

$$\chi_{k,j} \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T \pi_{k,j}(t)dt. \tag{4}$$

3. Chimera graph and related graphs

- An $N \times N$ Chimera graph, noted as C_N, consists of $8N^2$ vertices arranged as N^2 complete bipartite graphs $K_{4,4}$.
- Each unit cell consists of 8 vertices, 4 horizontal and 4 vertical. All vertices on the left are coupled to the vertices on the right and vice versa.
- In between cells, each vertex on the left is furthermore coupled vertically to the corresponding vertex in the unit cell above and below, while each of the ones on the right is horizontally coupled to the corresponding vertex in the unit cells to the left and right, see Fig. 1(a).
- Periodic boundary condition Chimera graph.
- Weak-strong cluster graph, see Fig. 1(b).

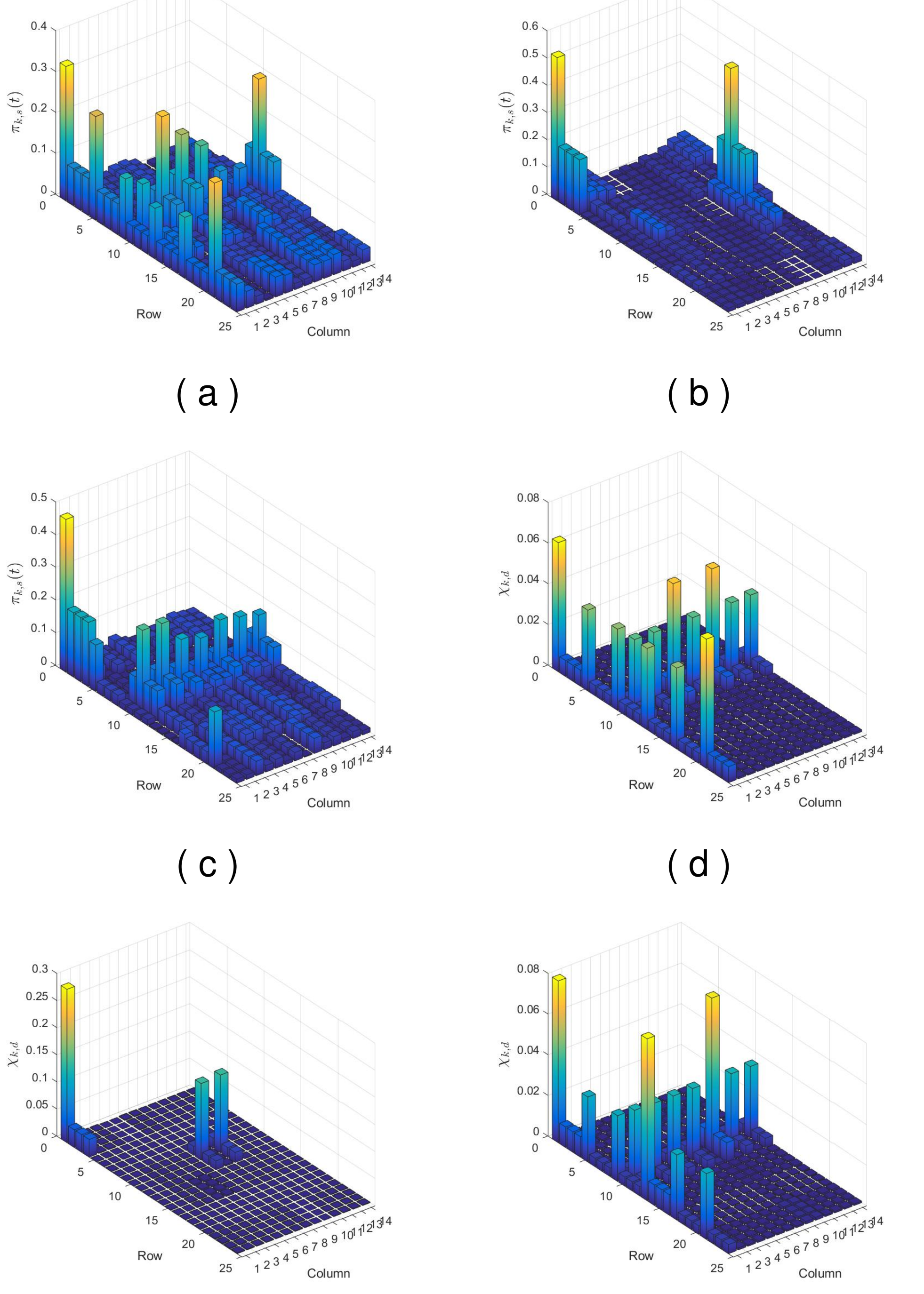


(a) (b)

Figure 1: *The structure of (a) Chimera graph and (b) Weak-strong cluster graph.*

4. The behavior of the walker

- If the walker is located on the left side vertex of the cell at the initial moment, then after a period of walking, the walker always occurs with a high probability at those vertices which is connected with the initial vertex with a red line.
- If the walker is located on the right side vertex of the cell at the initial moment, then after a period of walking, the walker always occurs with a high probability at those vertices which is connected with the initial vertex with a green line.
- If the walker is located on the superposition of left side vertex and right side vertex, the walker always occurs with a high probability at those vertices which is connected with the initial superposition vertex with a red line or a green line.
- If the initial position is not directly connected to another vertex with the red line or the green line, the walker will be localized, see Fig. 2(b).
- Here we give some snapshots of the probabilities $\pi_{k,s}$ to be at time $t = 6$ at vertex k when starting at vertex $s = \frac{1}{\sqrt{2}}[(1,1)+(10,10)]$, see Fig. 2(a), 2(b), 2(c), Here time is given in units of γ^{-1}.

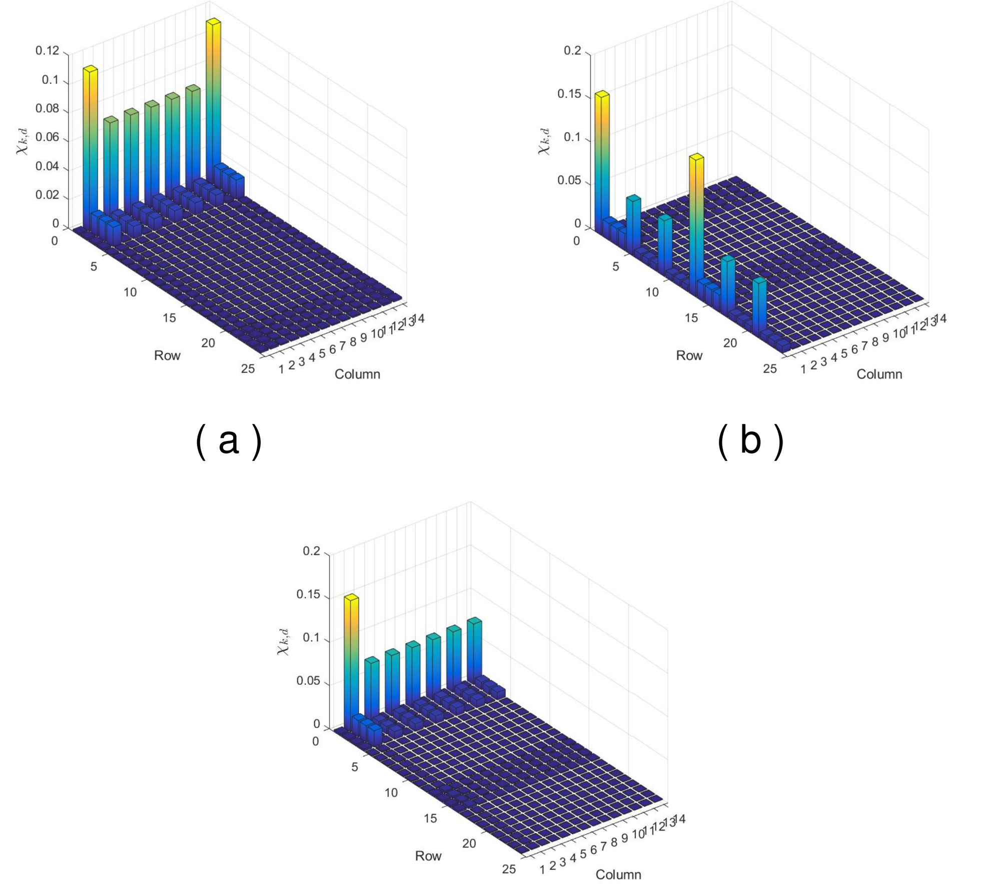


(a) (b) (c) (d) (e) (f)

Figure 2: *(a)Snapshots of $\pi_{k,s}(t = 6)$ on chimera graph. (b)Snapshots of $\pi_{k,s}(t = 6)$ on weak-strong cluster graph. (c)Snapshots of $\pi_{k,s}(t = 6)$ on periodic boundary condition chimera graph. (d)Snapshots of $\chi_{k,s}$ on chimera graph. (e)Snapshots of $\chi_{k,s}$ on weak-strong cluster graph. (f)Snapshots of $\chi_{k,s}$ on periodic boundary condition chimera graph.*

5. Limiting probabilities

- Starting in one vertex, the walker is distributed in units connected with green line or red line depending on the initial position directly connected with the green line or the red line, and will be localized if neither case is satisfied.
- For open boundary chimera graph, the limiting probability distribution pattern is mirror symmetric due to the symmetry of chimera graph, shown in fig. 3(a)
- For periodic boundary chimera graph, if the row number is a multiple of 8 and the walker initialized in the left side or the column number is a multiple of 4 and the walker initialized in the right side, the limiting probability distribution has periodicity, shown in fig. 3(b). Otherwise, periodicity does not occur, shown in fig. 3(c).



(a) (b) (c)

Figure 3: *(a)Snapshots of the limiting probabilities $\chi_{k,c}$ on open boundary chimera graph (b)Snapshots of the limiting probabilities $\chi_{k,c}$ on periodical boundary chimera graph with periodicity (c)Snapshots of the limiting probabilities $\chi_{k,c}$ on periodical boundary chimera graph without periodicity.*

6. Conclusion

- We study quantum walk on Chimera graph which is important for performing quantum annealing, and we explore the nature of quantum walks on variants of Chimera graph.
- Features of these quantum walks provide useful insight into the nature of the Chimera graph, including greater and lesser connectivity, isotropic spreading and localization.
- We analyze finite-size effects due to limited width and length of the graph. We explore the effects of different boundary conditions such as periodic and reflecting effects are explained via spectral analysis.
- The properties of stationary states and spectral analysis enables us to characterize asymptotic behavior of the quantum walker.

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