About the weak-strong cluster problem and its solvers

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1. Introduction

- D-Wave quantum machine is the first commercially available quantum machine over the world. Many attempts have been made to exploit whether there is a quantum speedup in the machine, while until now the answer to it is still elusive. It has been pointed out [1] that quantum annealing might not have better performance of finding the ground state than the classical corresponding algorithm over the spin glass Ising problem, in which the coupling strength of spins is randomly chosen from -1 or 1.
- So to show the advantage of the quantum finite range tunneling in the computational process, a carefully drafted problem, namely the weak-strong cluster problem [2], was proposed as a benchmark problem. This specific problem has tall and narrow energy barriers separating local minima, so D-Wave 2X can achieve significant runtime advantage over the the classical sequential optimization methods. While, because of the sparse connectivity of the D-Wave 2X architecture, some tailored classical algorithms, like Hamze-de Freitas-Selby(HFS) algorithm[3, 4] may excel in the scaling comparisons [5]. Here we will give a brief introduction to the problem they studied and its quantum and classical solvers, and with the comparison results, we can have a better understanding of what this quantum machine's computational value.

2. Weak-strong cluster problem

- The weak-strong cluster model is one of the tailored problem for the quantum optimizers. Here we introduce the weak-strong clusters of 16 qubits in two unit cells. And each cluster can coincide with a unit cell of the native hardware Chimera graph. In Fig.1, the spins in the right cluster are coupled to the "strong" external field ($h_2 = -1$). While, "weak" means the coupled external field in the left is weak. Usually we take $h_1 = 0.44 < \frac{J}{2}$. The four couplings between the two clusters are the same, $J = 1$ or $J = -1$, correspongding to the ferromagnetic or antiferromagneticare coupling.

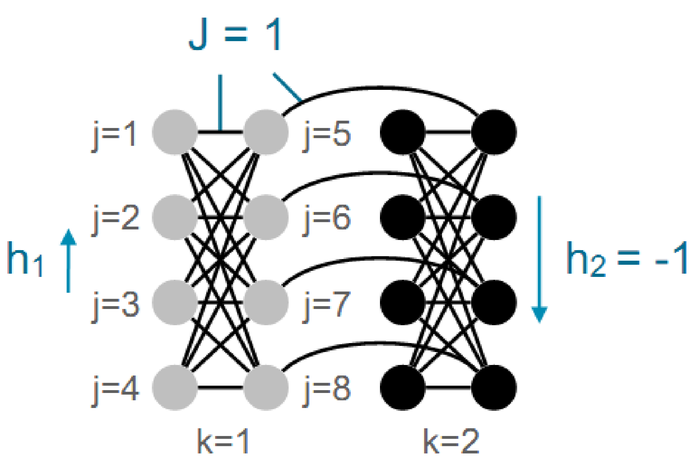

Figure 1: *A pair of weak-strong clusters. The right eight spins(in the dark dots) in the right are coupled with the external weak field. The left eight spins(in the grey dots) are coupled with the external strong field. And the inner coupling is set to $J = 1$.*

- The Hamiltonian of the system is in Ising form

$$H_P = H_1 + H_2 + H_{1,2} \tag{1}$$

$$H_k = -J \sum_{\langle j,j' \rangle \in intra} \sigma^z_{k,j}\sigma^z_{k,j'} - \sum_{j=1}^{8} h_k \sigma^z_{k,j}, k \in \{1,2\} \tag{2}$$

$$H_{1,2} = -J \sum_{j \in inter} \sigma^z_1 \sigma^z_{2,j} \tag{3}$$

- To make the problem bigger, each weak-strong cluster can be used to build up the "weak-strong cluster networks" problem in a glassy fashion, which means the coupling between the neighbouring strong cluster will be randomly chosen from +1(ferromagnetic) or -1(antiferromagnetic).

3. Quantum Anealing

- D-Wave 2X is a quantum annealer that make the use of the quantum annealing to solve the optimization problems. Many problems can be formed in the Ising formulation. For example, a class of problems called quadratic unconstrained binary optimization(QUBO) problem can be embeded to the hardware topology with the help of the minor-embedding technology
- Here is the model of the quantum annealing. With the assistance of the x-direction tranverse magenetic field, the time-dependent quantum Hamiltionian

$$H(t) = -A(t)H_D + B(t)H_P, t \in [0, t_a] \tag{4}$$

- Where H_P is the Hamiltonian of the problem we want to solve. And $H_D = \sum_i \sigma^x_i$ is the driver Hamiltonian, which can flip the qubit and provide a source for quantum fluctuations. So the annealing schedule will start with the tranverse field term(i.e., $B(0) = 0$) whose ground state is easily constructed, and end up with our problem term(i.e., $A(t_a) = 0$) whose ground state is what we want. During the annealing time, $B(t)$ will increase meanwhile $A(t)$ will decrease.
- The solving process the the quantum annealing process, which is a quantum analog of the classical annealing process. The difference between them is that classical annealing increases the temperature to hop up the solution escaping from the local optimal solution to the global solution, while the quantum annealing using the quantum tunneling to go through the energy barriers to get the global optima. There are some discussions about whether the D-Wave quantum annealer operates in the quantum regime or in the classical evolution in these papers.

4. Hamze-de Freitas-Selby algorithm

- Hamze-de Freitas-Selby(HFS) algorithm is a subgraph-based sampling algorithm. When sampling from the Gibbs distribution, instead of updating one spin according to its immediate neighbours at each run in the Markov chain Monte Carlo(MCMC), Selby used an induced subgraph update method, which can be more efficient.

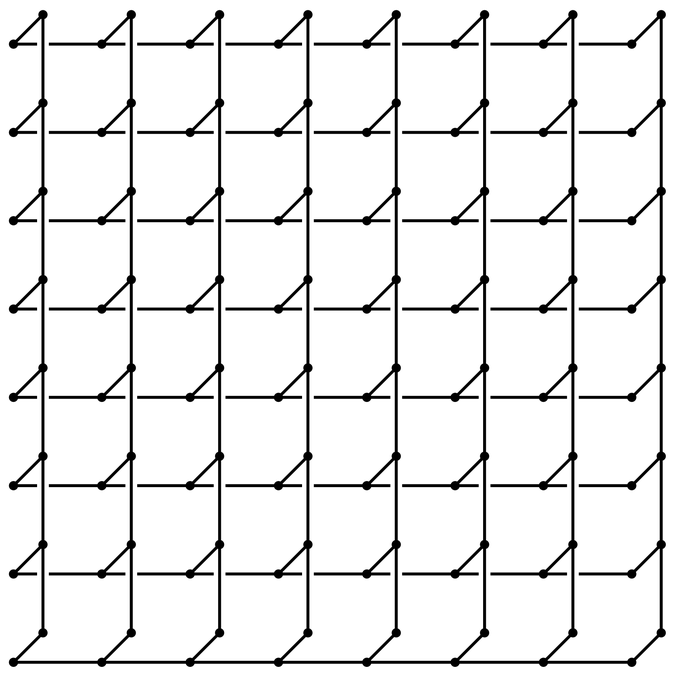

Figure 2: *The Chimera graph with 8×8 big vertices. Every four vertices in one side in Fig. 1 is combined as a big vertex in this graph.*

- Given a graph G, we will get a collection of induced subgraphs $T_1, ..., T_m$ that contain all edges of G. Another restriction is that $\cup T_i = G$ so that every vertex and edge is represented in some T_i.
- The idea is that the subgraph T_i of the graph should be easily to solve exact. A good choice of the induced subgraph could be the subgraphs of a given treewidth. Treewidth of a graph is the minimum width over all possible tree-decompositons of the graph. When treewidth is 1, that means the subgraph is a tree.
- So given an induced subgraph, T, and a spin configuration, $\mathbf{S}_{G\setminus T} = \{S_i \mid i \in G \setminus T\}$ defined on the reminder of G. So $P_T(\cdot \mid \mathbf{S}_{G\setminus T})$ is the probability conditioned on $\mathbf{S}_{G\setminus T}$.
- Then take T to be a random T_i from the subgraph collection and replace $\mathbf{S}_T$ with a random configuration chosen according to the conditional distribution $P_T(\cdot \mid \mathbf{S}_{G\setminus T})$. It can be proved this kind of sampling method will satisfy the detail balance[4].
- When dealing with the Chimera graph, he divided the bipartite graph $K_{4,4}$ into its native to part: the left four vertices as a big vertex, the same for the right four vertices. Fig. 2 shows the big vertices connections of the Chimera graph. Then the method mentioned above can be applied to this big vertices graph.
- He showed the advantage over the single-site-update-based sampling by testing different subgraphs with treewidth of 1 and 2 combined with the ground state finding method and the parallel tempering method.

5. Comparisons of the quantum and classical solvers

- In [2], they only compared the quantum annealing with its classical corresponding algorithm simulated annealing(SA)) and the Quantum Monte Carlo(QMC). And they claimed D-Wave machine can be faster than the QMC about eight orders of magnitude.
- In [5], they compared D-Wave 2X not only with SA and QMC, but also with other classical method, like the hybrid cluster method(HCM), the isoenergetic cluster algorithm(ICM), population annealing Monte Carlo, superspin heuristic(SS), and the HFS algorithm. They make the conclusion that while D-Wave 2X has a better scaling compared to SA and QMC, some other tailored and nontailored algothms can show a better asympototic scaling(see Fig. 3).

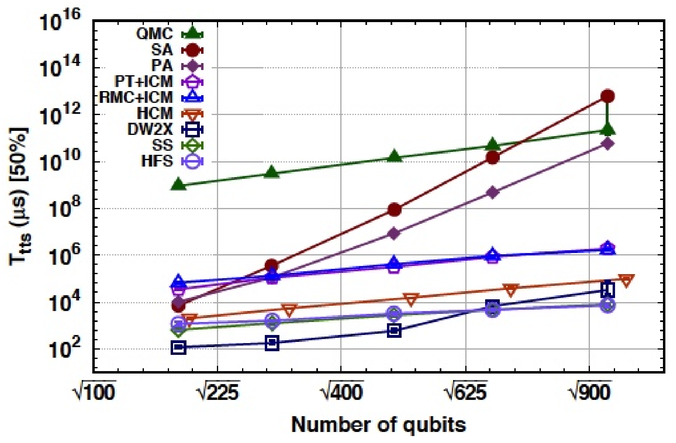

Figure 3: *The computational scaling comparion*

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This work is supported by National Laboratory for Physical Sciences at the Microscale Fellowship, China's 1000 Talent Plan and National Natural Science Foundation of China.