Coincidence landscapes for polarized bosons [1]

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Abstract

Passive optical interferometry with single photons injected into some input ports and vacuum into others is enriched by admitting polarization, thereby replacing the scalar electromagnetic description by a vector theory, with the recent triad phase being a celebrated example of this richness. On the other hand, incorporating polarization into interferometry is known to be equivalent to scalar theory if the number of channels is doubled. We show that passive multiphoton m-channel interferometry described by SU(m) transformations is replaced by SU$(2m)$ interferometry if polarization is included and thus that the multiphoton coincidence landscape, whose domain corresponds to various relative delays between photon arrival times, is fully explained by the now-standard approach of using immanants to compute coincidence sampling probabilities. Consequently, we show that the triad phase is manifested simply as SU(6) interferometry with three input photons, with one photon in each of three different input ports. Our analysis incorporates passive polarization multichannel interferometry into the existing scalar-field approach to computing multiphoton coincidence probabilities in interferometry and demystifies the triad phase.

1. Motivation

Typically, multiphoton interference is treated as a scalar field theory: polarization is ignored. Ignoring polarization is not detrimental for studying multiphoton interferometry as polarization can rather trivially be converted to a path degree of freedom. Polarization is included in m-channel interferometry by combining a U(2) transformation with a U(m) transformation, resulting in U$(2m)$ irreps using the subchain

$$\mathrm{U}(2m) \supset \mathrm{U}(m) \times \mathrm{U}(2) \tag{1}$$

transformation and the dual pair of U(2) and U(m) when the same U(2) polarization transformation acts on all spatial modes [2]. Dhand and Goyal decomposed arbitrary unitary transformations for photonic states in terms of spatial and internal modes [3]. Despite a mathematical equivalence between polarization and path degrees of freedom, some confusion has arisen about the sufficiency of mutual photon distinguishability in explaining the features of coincidence rates. A chief goal of our work is to show that polarization degrees of freedom, independent for each physical mode, are trivially absorbed into an SU$(2m)$ transformation.

2. Group structure in the multiphoton interference

Two-photon interference has been studied extensively since the celebrated 1987 Hong-Ou-Mandel (HOM) experiment [4]. In the quantum description, the HOM dip corresponds to a null coincidence for zero relative time delay: the quantum mechanical description forbids each detector to see a photon when they are indistinguishable. For long delays relative to the duration of the photonic wave packet, each photon has a 50% chance of being reflected or transmitted so the coincidence probably is one-half.

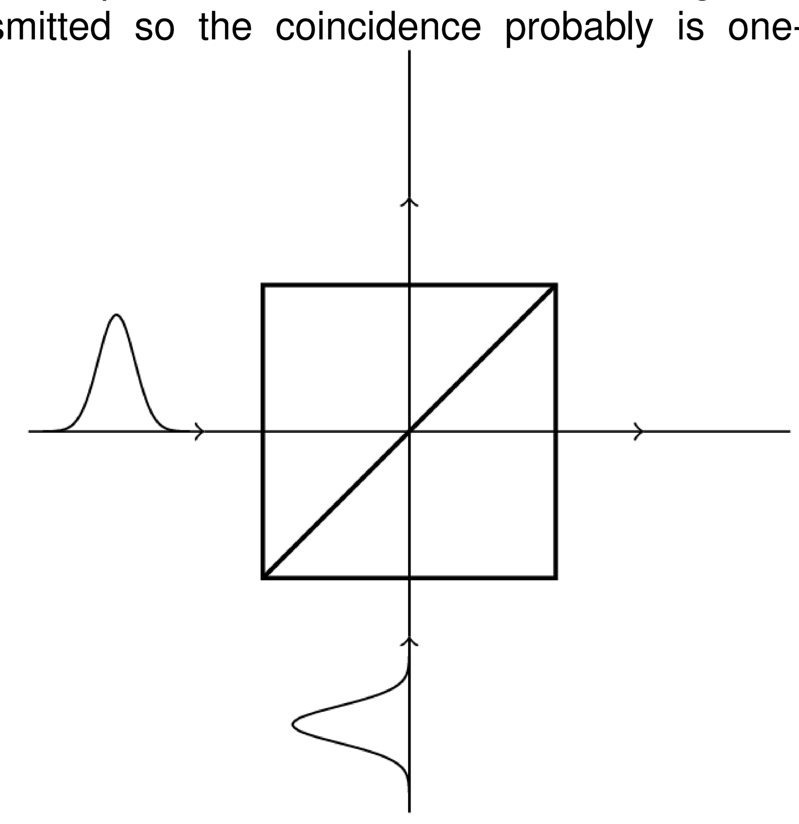

Figure 1: *Two-photon interference at a beam-splitter*

These two cases can be unified in the following Young diagram:

$$\square \otimes \square = \square\square \oplus \begin{smallmatrix}\square\\ \square\end{smallmatrix} \tag{2}$$

When photons are indistinguishable, the main contribution comes from the irreducible subspace $\square\square$, which means two photons are symmetric under the permutation S$_n$. In the other case, there are both contributions from $\square\square$ and $\begin{smallmatrix}\square\\ \square\end{smallmatrix}$.
More specifically, if we assume the input photonic state at each port as:

$$|\Psi_{\text{in}}\rangle = \int \mathrm{d}\omega_1 \phi(\omega_1) \mathrm{e}^{-\mathrm{i}\tau_1\omega_1} a_1^\dagger(\omega_1) \int \mathrm{d}\omega_2 \phi(\omega_2) \mathrm{e}^{-\mathrm{i}\tau_2\omega_2} a_2^\dagger(\omega_2) \tag{3}$$

where τ is the time delay and $\phi(\omega)$ is the frequency spectral.
With the measurement:

$$M = \int \mathrm{d}\omega_1 \omega_2 a_1^\dagger(\omega_1) a_2^\dagger(\omega_2) |0\rangle \langle 0| a_1(\omega_1) a_2(\omega_2), \tag{4}$$

the coincidence rate detected at the two output ports with each occupied by one photon is:

$$C = \langle \Psi_{\text{out}} | M | \Psi_{\text{out}} \rangle = \boldsymbol{\nu}^\dagger \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + |\beta_{12}(\tau_1,\tau_2)|^2 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \boldsymbol{\nu} \tag{5}$$

with $|\Psi_{\text{out}}\rangle = U |\Psi_{\text{in}}\rangle$, $\boldsymbol{\nu}^\dagger = \frac{1}{\sqrt{2}} (\text{perm}U \ \det U)$ and U is the scattering matrix of the interferometer. When the spectrum overlap β between two photons is unity, i.e., the coincidence is invariant under permutation of these two photons, the result only contains the permanent.
Above form gives us a systematic generalization from the two-photon to high-order coincidence dips or peaks arises from treating such scattering transformations as elements of a unitary group [5–8]. With Schur-Weyl duality between the symmetric group S$_n$ of n objects and the unitary group U(m) yields an immanant-based formalism that makes coincidence landscapes amenable to interpretations based on permutational symmetries of photons described by S$_n$ [9–12]. Like the permanent accounting for the indistinguishablity of photons, immanants naturally account for the partial distinguishability of photons.
For the multiphoton case, the Young diagram equation can also give us the direct way of decomposing the many-body Hilbert space into each irreducible subspace. Here we take the three photons as an example:

$$\square \otimes \square \otimes \square = \square\square\square \oplus \begin{smallmatrix}\square\square\\ \square\end{smallmatrix} \oplus \begin{smallmatrix}\square\square\\ \square\end{smallmatrix} \oplus \begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix} \tag{6}$$

In this case, the coincidence rate can be written:

$$C = \boldsymbol{\nu}^\dagger R(\boldsymbol{\tau}) \boldsymbol{\nu} \tag{7}$$

with R called the rate matrix, a linear combination of matrices of permutation operators. The weight of each permutation matrix is given by the overlap between the state and the corresponding permuted state. When R is block diagonalized by the conjugate class operator of S$_n$, each ν_i will be a linear combination of the immanant in the corresponding S$_n$ irreducible representation, which is defined as

$$\text{imm}^{\{\lambda\}} U = \sum_\sigma \chi^{\{\lambda\}}(\sigma) \prod_i U_{i\sigma(i)} \tag{8}$$

As an example, when the photon number is 3, we can see $\boldsymbol{\nu}$:

$$\begin{pmatrix} \text{perm}U \\ \det U \\ \frac{1}{2\sqrt{3}} \left(\text{imm}^{2,1} U + \text{imm}^{2,1} U_{312} \right) \\ \frac{1}{6} \left(\text{imm}^{2,1} U - 2\text{imm}^{2,1} U_{132} - \text{imm}^{2,1} U_{213} + 2\text{imm}^{2,1} U_{312} \right) \\ \frac{1}{6} \left(\text{imm}^{2,1} U + 2\text{imm}^{2,1} U_{132} + \text{imm}^{2,1} U_{213} + 2\text{imm}^{2,1} U_{312} \right) \\ \frac{1}{2\sqrt{3}} \left(-\text{imm}^{2,1} U + \text{imm}^{2,1} U_{312} \right) \end{pmatrix} \tag{9}$$

3. From SU(m) to SU$(2m)$

When the polarization of photons is taken into consideration, we can visually decompose the scattering matrix into the polarization P and path scattering T parts:

$$U = TP \tag{10}$$

with T the double of the original path scattering matrix $\tilde{T}$ as:

$$T = \begin{pmatrix} \tilde{T} & 0 \\ 0 & \tilde{T} \end{pmatrix} \tag{11}$$

and

$$P = \oplus_{i=1}^m P_{2\times 2}^{(i)} \tag{12}$$

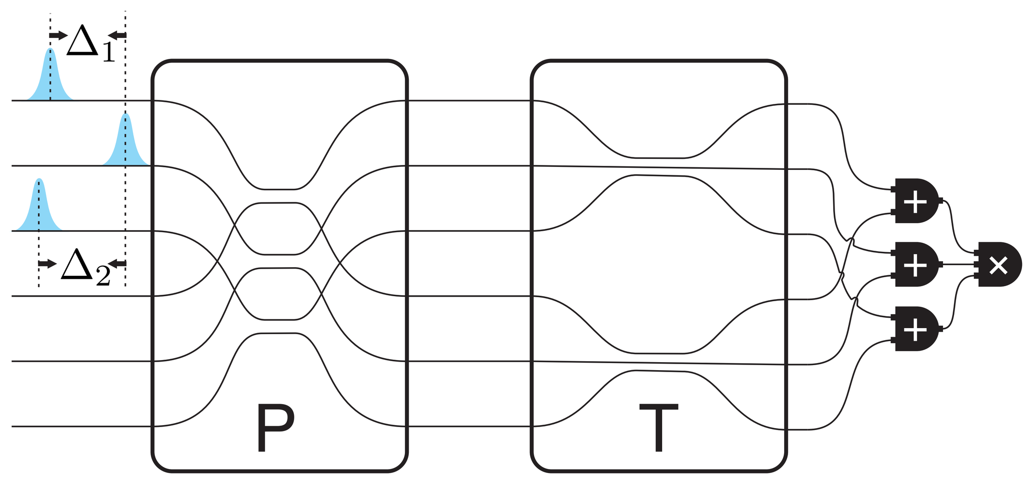

Figure 2: *Decomposition of interference network and setup for the detectors.*

The coincidence rate is different in different measurement settings depending on whether the detectors are polarization-sensitive or not. For the former case, the polarization degree of freedom then natually be absorbed as a path degree of freedom in the scattering matrix. Then the interferometer can be described as SU$(2m)$ instead of SU(m). We suppose the polarization is adjusted by the parameter $\boldsymbol{\theta}$, then the coincidence rate is

$$C = \boldsymbol{\nu}(\boldsymbol{\theta})^\dagger R(\boldsymbol{\tau}) \boldsymbol{\nu}(\boldsymbol{\theta}) \tag{13}$$

Othersize, the polarization has the same effect as the time delay to change the photon's indistinguishability in the rate matrix:

$$C = \boldsymbol{\nu}^\dagger R(\boldsymbol{\tau}, \boldsymbol{\theta}) \boldsymbol{\nu} \tag{14}$$

As the permutation matrices are not affected by the introduction of the polarization degree of freedom, block diagonalization of $R(\boldsymbol{\theta}, \boldsymbol{\tau})$ yields the coincidence rate in terms of immanants. In a generalization of previous results, the distinguishability of a photon is not limited to the temporal overlap of pulses but also include the polarization overlap.

4. Conclusion

We have described a model for polarized multiphoton interference in a generalized scattering interferometer, in which the detectors are either polarization sensitive or insensitive. In our model, polarization is merged into the transformation network by doubling the ports to realize an SU$(2m)$ transformation. By changing polarizations, we consequently change the scattering matrix.
Specifically, for polarization-insensitive detectors, the polarization is merged into our rate matrix, then changing the polarization is equivalently changing the photons' distinguishabilities. We have shown that the two-photon pairwise distinguishabilities, which include the information about the independent parameters $\boldsymbol{\tau}$ and $\boldsymbol{\theta}$, already suffice to describe the multiphoton interference.

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